Quantum simulation

Spectroscopic signatures of localization with interacting photons in superconducting qubits

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Quantized eigenenergies and their associated wave functions provide extensive information for predicting the physics of quantum many-body systems. Using a chain of nine superconducting qubits, we implement a technique for resolving the energy levels of interacting photons. We benchmark this method by capturing the main features of the intricate energy spectrum predicted for two-dimensional electrons in a magnetic field—the Hofstadter butterfly. We introduce a disorder to study the statistics of the energy levels of the system as it undergoes the transition from a thermalized to a localized phase. Our work introduces a many-body spectroscopy technique to study quantum phases of matter.

Consider a system of interacting particles isolated from the environment initially prepared in a very low entropy state far from equilibrium. It is often observed that the system acts as its own thermal reservoir and approaches the equilibrium state. In this thermal phase, the system shows ergodic behavior, wherein it uniformly explores all accessible states over time. Recent works discuss the emergence of another phase in certain parameter regimes in which ergodicity breaks down and thermal equilibrium becomes unattainable (1–8). This phase is referred to as the many-body localized (MBL) phase (9–16). The conventional quantum phase transitions, e.g., from para- to ferromagnetic, are characterized by changes in the ground state of the system. However, the signature differences between the thermalized and MBL phases are in dynamical behaviors, indicating that the transition involves change in the properties of all many-body eigenstates of the system. Hence, the physics goes beyond the ground state and requires study of the entire energy spectrum, which constitutes an experimental challenge.

In quantum physics, the quantized eigenenergies and their associated wave functions provide extensive information for predicting the chemistry of molecules or physics of condensed-matter systems. Creating local perturbations and recording the subsequent response of the system as a function of time can reveal the characteristic modes of that system (17, 18). Our method for measuring the energy spectrum of a Hamiltonian is based on this principle. For fixed Hamiltonians, the state of a system evolves according to Schrödinger equation

$$\psi(t) = \sum_\alpha C_\alpha e^{-iE_\alpha t/\hbar} |\phi_\alpha\rangle$$

where $E_\alpha$ is an eigenenergy of the Hamiltonian and $|\phi_\alpha\rangle$ is the corresponding eigenstate. Equation 1 implies that $\{E_\alpha\}$ and $\{C_\alpha\}$ determine the frequencies and the amplitudes of the modulations in $|\psi(t)\rangle$, respectively. The similarity of Eq. 1 and a Fourier transform (FT) relation suggests that the frequencies observed in the FT of the evolution could in principle reveal $\{E_\alpha\}$. In addition, the magnitudes of FT terms provide $\{C_\alpha\}$; these coefficients set the relative contribution of each $|\phi_\alpha\rangle$ to a given dynamics.

Using nine superconducting qubits (Fig. 1A), we constructed a one-dimensional (1D) lattice of bosons and implemented a spectroscopy method based on the above-mentioned fundamental postulate of quantum mechanics. Each of our qubits can be thought of as a non-linear photonic resonator in the microwave regime (19, 20). The Hamiltonian of the chain can be described by the Bose-Hubbard model

$$H_{BH} = \sum_{n=1}^{9} \mu_n a_n^\dagger a_n + \frac{U}{2} \sum_{n=1}^{9} a_n^\dagger a_n (a_n^\dagger a_n - 1)$$

$$+ J \sum_{n=1}^{8} a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1}$$

where $a^\dagger$ ($a$) denotes the bosonic creation (annihilation) operator, $\mu_n$ is the on-site potential, $J$ is the hopping rate of the photons between nearest-neighbor lattice sites, and $U$ is the on-site interaction. The qubit frequency, the nearest-neighbor coupling, and nonlinearity set $\mu_n$, $J$, and $U$, respectively. In our system, we can vary the first two on nanosecond time scales, but $U$ is fixed.

In Fig. 1, we show how to identify the eigenenergies of Eq. 2 when it describes the hopping of a single photon in a disordered potential. The disordered potential is realized by choosing a random number from a uniform distribution $[-\Delta, \Delta]$ for each lattice site, and detuning the qubits accordingly. Initially, there is no photon in the system and all the qubits are in $|0\rangle$ state. Then, we place the nth qubit $Q_n$ in the superposition of $|0\rangle$ and $|1\rangle$ (Fig. 1B). We measure the evolution of $|\psi\rangle = (c_0^\dagger + c_1^\dagger) |\psi\rangle$, where $c^\dagger$ and $c^\dagger$ are Pauli operators (acting on the $|0\rangle$ and $|1\rangle$ subspace) (Fig. 1C). From the $|c_0^\dagger\rangle$ and $|c_1^\dagger\rangle$ measurements, we construct $\langle j, n \rangle = \langle c_0^\dagger | c_1^\dagger \rangle$. Next, we vary $n$ from 1 to 9 to ensure that the energy spectrum is fully resolved. By varying $n$, the initial states form a complete basis, and then every energy eigenstate is certain to have some overlap with one of the initial states and hence can be detected. Figure 1D shows the square of FT amplitudes of $|c_j\rangle(n)$ for each $Q_n$ in which distinct peaks can be readily identified. The result of averaging the squared FT amplitudes over nine different initial states is depicted in Fig. 1E, where nine major peaks appear; their frequencies are the nine eigenenergies of the Hamiltonian. The particular choices of initial states and the observables are made to avoid appearance of undesired energy peaks in the spectrum (17, 18, 21).

Next, we demonstrate our capability to accurately set the terms in a specific Hamiltonian and resolve the corresponding eigenenergies. We simulate the problem of Bloch electrons on a 2D lattice subject to a perpendicularly applied magnetic field $B$. The magnetic length ($l_B = \sqrt{\hbar/eB}$) and lattice constant $a$ characterize the electron’s motion, and their interplay sets the physics. The resulting energy spectrum was first calculated by Hofstadter and resembles a butterfly (22). For typical crystals, the magnetic field required to “squeeze” one flux quantum through the unit cell is of the order of several tens of thousands of tesla, too high to be experimentally feasible. Recently, some features associated with the Hofstadter’s butterfly were experimentally realized by using superlattices in graphene and cold-atom systems (23–28).

The Hofstadter energy spectra can be parameterized by a single dimensionless magnetic field, $b = a^2 eB/\hbar$, which counts the number of magnetic flux quanta per unit cell. In the tight binding approximation, the effective Hamiltonian is the 1D Harper Hamiltonian (22)

$$H_{Harper} = \Delta \sum_{n=1}^{9} \cos(2\pi nb)a_n^\dagger a_n$$

$$+ J \sum_{n=1}^{8} a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1}$$

$$H_{Harper}$$ is a special case of $H_{BH}$, reached by setting $\mu_n = 3\cos(2\pi nb)$ and exciting only one photon in the system, i.e., the interaction term is zero. In this limit, whether the particles are fermionic or bosonic has no influence on the...
We realize a quasi-periodic disorder potential by using the following procedure (two-photon protocol). After the evolution, a microwave \(\pi/2\) pulse is applied to the qubit to measure \(\langle \sigma_z^i \rangle \) or \(\langle \sigma_z^i \rangle\).

(C) Typical data set showing \(\langle \sigma_z^i \rangle\) and \(\langle \sigma_z^i \rangle\) versus time. (D) The FT of \(\chi_2(n) = \langle \sigma_z^i \rangle + i \langle \sigma_z^i \rangle\) for \(n \in \{1, 2, \ldots, 9\}\). The peaks in the FT correspond to the eigenvalues of the Hamiltonian. The probability of a Fock state on \(Q_6\) being in the ninth eigenstate \(P_{9,6}\) is highlighted. (E) Average of the square of FT amplitudes shown in (D). Averaging is done to show all nine peaks in one curve.

behavior of the system. In Fig. 2, we vary \(b\) from 0 to 1 and realize 100 different instances of \(H_{\text{on}}\), similar to Fig. 1. For each \(b\) value, initial states with the \(n\)th qubit excited are created, the evolution of \(\langle \sigma_z^i \rangle\) and \(\langle \sigma_z^i \rangle\) is measured, and \(n\) is varied from 1 to 9. For each \(b\) value, Fig. 2A shows the square magnitude summation of the FT of \(\{\chi_2(n)\}\).

For large lattices, it is known that for rational \(b\), all energy bands split into subbands, and for irrational \(b\), the spectra become fractal and form a Cantor set. Because there are only nine sites, what is seen in Fig. 2A are the remnants of those bands. Nevertheless, the overall measured spectrum still resembles a butterfly. We focus on this pattern of level crossings and meanderings and ask how well the measurements match simulation. In Fig. 2B, we present the numerically computed eigenenergies with solid gray lines and the measured peaks from Fig. 2A with colored dots (21). The color of the dots shows the distance in megahertz of the peaks from the simulation values. The deviations are mainly systematic calibration errors, and occasionally an erroneous peak is identified in the subroute. The average deviation is 3.5 MHz. This implies that we can set the matrix elements of the Hamiltonian, which in this case includes 17 terms, with <2% error. This capability of controlling a large quantum system is achieved through careful modeling of the qubits as nonlinear resonators.

By placing two photons in the system, we next study the simplest interacting bosonic cases \((U=0)\), with no mapping to electronic system. The rest of the data presented in this work are obtained by using the following procedure (two-photon protocol). We realize a quasi-periodic disorder potential by setting \(\mu_q = \Delta \cos(2\pi nb)\). In total, four different irrational values of \(b\) are randomly chosen from \([0,1]\), and the corresponding results are averaged. The irrational choice of \(b\) ensures that the periodicity of the potential and lattice are incommensurate. In Eq. 2, we set \(J/2\pi = 50\) MHz, which results in \(U/J = 3.5\). The initial states are made by placing two qubits \((Q_n \text{~and~} Q_m)\) in the superposition of the \(|0\rangle\) and \(|1\rangle\) states. We measure two-point correlations and construct \(\chi_2(n,m) = \langle \sigma_z^i \sigma_z^m \rangle + i \langle \sigma_z^i \sigma_z^m \rangle + i \langle \sigma_z^i \sigma_z^m \rangle\). The peaks observed in the FT of \(\chi_2(n,m)\) are the eigenenergies of \(H_{\text{on}}\) in the two-photon manifold (21).

Perhaps the most direct way of examining ergodic dynamics and its breakdown is by studying the distribution of the energy levels (29–31). Using the two-photon protocol, we measure the evolution of \(\chi_2(n,m)\) for various strengths of disorder \(\Delta\). We identify the peaks in the FT of \(\chi_2(n,m)\) as the energy levels \(E_n\) (Fig. S3). Let \(s_n = E_{n+1} - E_n\) be the nearest-neighbor spacings (Fig. 3A), and level separation uniformity \(r_n = \min(s_n, s_{n-1}) / \max(s_n, s_{n-1})\). From our measured \(E_n\) we compute the associated \(r_n\) and construct their probability distribution (PD) (Fig. 3B). For low disorder, the PD is mainly centered around the \(r_n\) values close to half, and with an increase of disorder, the histogram’s peak shifts toward smaller \(r_n\) values.

It has been postulated that in the ergodic phase, the statistics of energy levels is the same as the ensemble of real Hermitian random matrices, which follow the Gaussian orthogonal ensemble (GOE) (31). In the many-body localized phase, the energy levels become uncorrelated owing to disorder, and hence a Poisson distribution in energy landscape is expected. The probability distribution of \(\{r_n\}\) for the ergodic and many-body localized phases, respectively, is

\[
P_{\text{GOE}}(r) = \frac{27}{4} \frac{r + r^2}{(1 + r + r^2)^{3/2}}.
\]

In Fig. 3C, we focus on \(\Delta/J = 1\) and 5, showing the measured histograms with dots and the numerical simulations with solid lines (21). The dashed lines are plots of Eq. 4, providing the expected behavior in the thermodynamic limit (number of sites \(N_y \rightarrow \infty\)), and for limiting values of \(\Delta/J\). In contrast to these cases, the finite size of our chain results in features that can be seen in both data and simulation. When disorder is small, the energy eigenstates are extended across the chain (Fig. 4), and hence the energy levels repel each other. Consequently, there are strong correlations between the levels, and an equidistant distribution of levels would be favorable. When \(\Delta\) becomes larger, the eigenstates become localized in space and unaware of each other’s presence at a given energy, and level repulsion ceases. Therefore, the levels independently distribute themselves, showing a Poisson distribution in the energy landscape. The exact realization of Poisson distribution takes place only when \(J/\Delta \rightarrow 0\); in our case, \(J/\Delta = 0.2\), which is where the peak in the histogram appears \(r_n = 0.2\). Because the Poisson distribution is a signature of independent events, we conclude that the transition from ergodic to localized phase is associated with vanishing correlations in energy levels.
A key signature of the transition from ergodic to MBL phase is the change in the localization length of the system from being extended over the entire system to localized over a few lattice sites. This physics can be studied by measuring the entire system to localized over a few lattice length of the system from being extended over to MBL phase is the change in the localization probability ratios. (PR)

\[ PR_{\text{Space}}(\alpha) = 1 - \sum_n P_{\alpha,n}^2, \quad PR_{\text{Energy}}(\alpha) = 1 - \sum_n P_{\alpha,n}^2 \]

\[ (5) \]

\( PR_{\text{Space}} \) indicates the number of sites over which an energy eigenstate \( |\alpha\rangle \) has an appreciable magnitude. Similarly, \( PR_{\text{Energy}} \) measures how many energy eigenstates have a discernible presence on lattice site \( n \). Note that the first moments of the probability distributions are normalization conditions \( \sum_n P_{\alpha,n} = 1 \) and \( \sum_n P_{\alpha,n}^2 = 1 \).

Having demonstrated that we can fully resolve the energy spectrum of the two-photon energy manifold, we now extract \( \{P_{\alpha,n}\} \). In Fig. 4A, we plot \( PR_{\text{Space}} \) for various disorder strengths in the order of increasing energy. In this energy manifold, there are 36 single- (e.g., \( |001000000\rangle \)) and 9 double-occupancy states (e.g., \( |000020000\rangle \)), which gives \( \begin{pmatrix} 9 \\ 2 \end{pmatrix} \) = 45 energy levels. For low disorder \( \Delta/J < 1 \), \( PR_{\text{Space}} \) is about 8, indicating that almost all energy eigenstates are extended over the entire chain of nine qubit lattice sites. As the strength of disorder increases, the eigenstates with their energies close to the edge of the energy band start to shrink, whereas the eigenstates with energies in the middle of the band remain extended at larger disorders. This is similar to the Anderson localization picture, in which localization begins at the edges of the band, and a mobility edge forms (the yellow hue) and approaches the center of the band as disorder becomes stronger (32). The existence of the mobility edge in MBL has been theoretically questioned, and proper investigation of it requires going to larger systems and finite size scaling (34). Given that numerical exact diagonalization is limited to small systems, scaling up the experiment could shed light on this matter and the general understanding of MBL (33, 35).

In Fig. 4B, we plot the \( PR_{\text{Energy}} \), which shows that as the disorder becomes stronger, the number of eigenstates present at a given lattice site is reduced, indicating that eigenstates are becoming localized on lattice sites. Furthermore, with increasing disorder, the eigenstates are avoiding the edges of the chain, and more eigenstates are present toward the center of the chain. The changes in \( PR_{\text{Space}} \) and \( PR_{\text{Energy}} \) are the fastest near \( \Delta/J = 2 \), suggestive of a phase transition that has been smeared out owing to finite-size

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Fig. 2. Hofstadter butterfly. In Eq. 3, we set on-site potentials Δ/2π = 50 MHz and coupling J/2π = 50 MHz. (A) Data similar to Fig. 1D, averaged squared FT magnitude, are shown for 100 values of dimensionless magnetic field \( b \) ranging from 0 to 1. (B) For each \( b \) value, we identify nine peaks and plot their location as a colored dot. The numerically computed eigenvalues of Eq. 2 are shown as gray lines (21). The color of each dot is the absolute value of the difference between the measured eigenvalue and the numerically computed one.

Fig. 3. Level statistics in a disordered potential. In Eq. 2, we set hopping to \( J/2\pi = 50 \) MHz, which fixes \( U/J = 3.5 \). To obtain a disordered potential, we set \( \mu_n = -\cos(2\pi nb) \) with four different irrational values of \( b \in [0, 1] \) chosen and the results averaged over \( b \). (A) The schematic of energy levels shows how \( r_\alpha \) is defined. (B) The histogram of \( P(r) \) measured for various values of disorder \( \Delta/J \) is presented as a color plot. (C) The measured histogram \( P(r) \) of \( \{r_\alpha\} \) for \( \Delta/J = 1 \) and 5. The dashed lines are plots of \( P_{\text{Poisson}} \) and \( P_{\text{GOE}} \) according to Eq. 4, and the solid lines are numerical simulations (21). The change from the GOE toward the Poisson distribution is indicative of vanishing of level repulsion with increase in \( \Delta \).
ultimately limited by the frequency broadening; calculations will begin to fail. The technique pre-
with a few tens of qubits, where classical sim-
ulations can be extracted by directly measuring
dependent experiments. However, a decisive dis-
whose value has been established through in-
becomes intractable for classical computers as the
size of the system grows. From the magnitude of
that results from decoherence. For large systems,
level spacing becomes exponentially denser and the current approach needs to be revised;
promising methods are suggested in (35, 36).
Nevertheless, the valuable computational resource
that our platform offers resides in measuring the
dynamics of observables; those dynamics quickly
are associated with the MBL phase transition.

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Fig. 4. Participation ratio and mobility edges. In Eq. 2, we set
$\frac{b}{\sqrt{5} - 1/2}$, $J/2\pi = 50$ MHz, which results in $U/J = 3.5$. We
measure the evolution of $\chi_{n}(n, m) = (\sigma_{n}^{x} \sigma_{m}^{x}) + (\sigma_{n}^{y} \sigma_{m}^{y}) + i(\sigma_{n}^{z} \sigma_{m}^{z})$ for all pairs of
$n, m \in \{1, 2, \ldots, 9\}$ as a function of time for various strengths of
$\Delta$ from the magnitude of the peaks seen in the FT of the data,
the probabilities relating the positions of two-photon Fock states
to energy eigenstates $\{P_{n,m}\}$ are extracted. See fig. S3 for details.
On the basis of these data, we calculated (A) $PR_{\text{Space}}$ and
(B) $PR_{\text{Energy}}$ using Eq. 5 and plotted the results. The $E_{\text{max}} - E_{\text{min}}$ is the
width of the energy band at a given $\Delta$.

effects. Nevertheless, we emphasize that the quantum phase transition to the MBL phase is
only defined in the thermodynamic limit ($N_{f} \rightarrow \infty$) (15). Given the finite size of our system and
the presence of only two interacting particles, it is interesting that we see finite-size precursors asso-
ciated with the MBL phase transition.

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Putting photons to work

Interacting quantum particles can behave in peculiar ways. To understand that behavior, physicists have turned to quantum simulation, in which a tunable and clean system can be monitored as it evolves under the influence of interactions. Roushan et al. used a chain of nine superconducting qubits to create effective interactions between normally noninteracting photons and directly measured the energy levels of their system. The interplay of interactions and disorder gave rise to a transition to a localized state. With an increase in the number of qubits, the technique should be able to tackle problems that are inaccessible to classical computers.

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