In advanced field theories, there can be more than four dimensions to space, the excess dimensions described as compacted and unobservable on everyday length scales. We report a simple model, unconnected to field theory, for a compacted dimension realized in a metallic metasurface periodically structured in the form of a grating comprising a series of singularities. An extra dimension of the grating is hidden, and the surface plasmon excitations, though localized at the surface, are characterized by three wave vectors rather than the two of typical two-dimensional metal grating. We propose an experimental realization in a doped graphene layer.

A conventional two-dimensional (2D) object is characterized by two quantum numbers. For example, the frequencies of surface plasmons on a periodic surface are labeled by the components of their momentum projected onto the surface axes. We describe theoretically systems that instead require three quantum numbers to label them: the two conventional in-plane momenta plus a third momentum corresponding to a compacted dimension hidden from view inside a singularity. Compacted dimensions are ingredients of advanced string theories (1, 2) in which the extra dimensions in a 4 + N dimensional theory are said to be compacted and so not directly observed on everyday length scales. As far as we know, our singular surfaces are the only physically realizable model of this curious effect. We give two instances of how this might be done.

We make use of the technique of transformation optics (3–5), which exploits the invariance of Maxwell’s equations under a coordinate transformation: Only the values of $\varepsilon$, $\mu$ are affected by the transformation. We used this theory to compact a dimension through a singular transformation that compresses one of the dimensions of a 3D system into one or more singular points. An example of the process is given (Fig. 1) for a 3D system (Fig. 1A), periodic in one of the dimensions and translationally invariant in the two other directions. The Fig. 1 blue-shaded areas are metallic and support surface plasmons (6) whose spectrum is characterized by three wave vectors: $k_x$, $k_y$, $k_z$, where $k_x$ is the wave vector heading out of the plane of the paper.

Once calculated in the initial frame can be found in the other frames through the properties of the transformation.

In the first step, we compress $x = -\infty$ to a point at the origin

$$\tilde{z} = \exp(z)$$  

which gives rise to Fig. 1B. This transformation produces knife edges that have been discussed previously (7), and Davis has commented on the continuous nature of the spectrum (8). Next, we compress $z' = +\infty$ to a point by inverting about $z'' = -a$, giving rise to a structure shown in Fig. 1C that is finite in the $x'', y''$ plane

$$z'' = \frac{1}{z'' + a}$$  

Last, we add a further transformation to create a periodic metasurface (Fig. 1D).

$$z'' = \frac{d}{2\pi \ln(z'' - \frac{1}{2a})}$$

The metasurface shown in Fig. 1D has some unusual properties. The modes of Fig. 1A are truly 3D, being spread out over the whole structure, and are characterized by the three wave vectors, $k_x$, $k_y$, $k_z$. By contrast, these modes when transformed through to Fig. 1D are now found to be exponentially localized at the interface and at first sight would seem to be 2D objects. However, this is not the case; the structure inherits the spectral characterization of the mother structure and therefore is labeled by the same three wave vectors, $k_x$, $k_y$, $k_z$. This affects the response of the metasurface to external stimuli. A conventional grating, free of any singularities such as sharp edges, is characterized by only two wave vectors, which are defined by the angle of incidence of external radiation. This leads to a discrete excitation spectrum. In other words, ordinary gratings appear colored to the eye. By contrast, external radiation incident on our metasurface defines only two of the three wave vectors, the third being selected by the frequency. Thus, the modes form a continuum and can be excited whatever the incident angle or frequency. Our singular metasurfaces are not colored; they are gray, or black in the limit of strong coupling, as we shall show.

For these transformed modes, radiation is captured in the broad smooth portions of the metasurface and travels toward the cusps, becoming increasingly compressed but never reaching the cusp. The effect of compression is to increase the field strength, which is inversely proportional to the local group velocity and in an ideal loss-free system would rise to infinity at the cusp. However, in a realistic system, losses would intervene and result in a finite but very large field enhancement, as in a surface-enhanced Raman scattering experiment.

In the following, we provide detailed calculations made by using the techniques of transformation optics (9), which we have successfully deployed on nonsingular gratings.
The dispersion of the modes for Fig. 1A is calculated assuming a Drude form for the metallic component.

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$  \hspace{1cm} (4)

The dielectric separating the metal layers is taken to be vacuum.

Shown in Fig. 2A is the dispersion of the modes at \(k_x = k_y = 0\) with respect to the hidden variable, \(k_z\), calculated for the structure in Fig. 1A. The ratio of metal to vacuum is 1:4, which makes the angle of the cusp in Fig. 1D to be 72°. The same calculation is shown in Fig. 2B extended to include \(k_y\) while \(k_x = 0\). There are two modes—one symmetric, the other antisymmetric—about the point of the cusp in Fig. 1D, the lower mode being dark.

The singularity harvests photons incident on the metasurface, concentrating their density at the tip. Shown in Fig. 3 are the phase and amplitude of the electric field at the metal dielectric interface for a mode with hidden variable \(k_x = 10\) at \(k_y = k_z = 0\). In a lossless system, the amplitude rises to infinity at the singular points, and the phase oscillates infinitely rapidly, resulting from compression of the infinite hidden dimension into a singular point. In a system with loss, this is still the case until a critical value of the loss is exceeded, at which point the amplification mechanism breaks down. The total energy content remains finite at all times.

In the second realization, we start from a thin slab of conductor, which could represent doped graphene, and use transformations previously reported (10) to transform the slab into a 2D grating (Fig. 4).

$$z' = \frac{d'}{2\pi} \log \left[ \frac{1}{2\pi(z+d)/d - \frac{i\omega}{\omega_0} + i\gamma} \right]$$  \hspace{1cm} (5)

where \(x_0, y_0, w_0\) are parameters that can be used to adjust the shape of the grating. Then, we add a further step, taking the limit at which the thickness of the grating vanishes at the minima marked by asterisks separated by period \(d\). The corresponding points marked in Fig. 4A are separated by \(d\). If we choose to keep the grating period and maximum thickness constant during the limiting process, this implies that \(d \to \infty\), and the asterisks recede to \(\pm \infty\). This implies that the whole of the continuous spectrum of the infinite \(y\) dimension in Fig. 4A is compressed into the finite segment length \(d'\) in Fig. 4E, the infinity vanishing into the singularity. At the same time, \(y'\) remains infinite in extent, so we have compressed two dimensions into one. The original \(y\) dimension and its spectrum are hidden in the singularities, and although Fig. 4E is still outwardly a 2D system, the spectrum is characterized by three wave vectors: \(k_y\) for the hidden dimension, \(k_x\) the new dimensions, and \(k_z\) the out-of-plane dimension. These steps comprise our second route to hidden dimensions through a different sort of singularity.

We propose an experimental realization: Doped graphene supports plasmonic excitations in the terahertz regime, and doping can be controlled by the proximity of a charged surface that attracts or repels electrons to or from that region of the graphene. In this way, graphene can be periodically doped to form a grating. Because graphene is very thin, the variation in conductivity can be modeled as a constant permittivity material varying in thickness and hence mapped onto the system illustrated in Fig. 4. Transformation optics has been shown to be a powerful tool for studying graphene (10), and as we have shown in (11), the grating can strongly couple incident radiation to plasmons, leading to strong absorption at the resonant frequency. In this sense, graphene patterned with a smooth grating is “colored”: absorption happens only at discrete frequencies. However, we can model our singular grating by increasing the modulation until the electron density, and therefore the conductivity, approaches zero at a singular point following the sort of profile shown in Fig. 4. Our simulations for...
a succession of narrowing gaps show a series of peaks in the transmissivity merging into a continuous absorption spectrum at the singularity.

We calculated the electromagnetic response of the graphene metasurface by considering a plane wave incident normal (k_y = k_w = 0) to the graphene layer with modulated conductivity and transforming it to a frame where graphene is homogeneously doped. The homogeneously doped graphene can be modeled as a thin slab of constant thickness (Fig. 4A). Then, we applied boundary conditions for the electromagnetic fields in the slab frame and included the radiative reaction of the graphene grating as it couples to external radiation. In order to approach the singularity smoothly, we used the transformation shown in Eq. 5. By allowing the grating minimum to approach a touching point, we can model periodic gratings that tend toward a singular metasurface.

Evolution of the transmissivity for three graphene metasurfaces as the singularity is approached is shown in Fig. 4, C to E (solid lines correspond to our analytical modeling based on transformation optics, and dots represent full electrodynamic simulations). In our calculations, we used the conductivity of graphene from the random phase approximation, which depends on frequency, chemical potential, mobility (10^4 cm² V⁻¹ s⁻¹) for the blue lines in Fig. 4, C to E), and temperature (T = 300 K). For all of them, the period is 2.5 μm, and the periodic conductivity profile peaks at 2.13 × 10⁻³ ohm⁻¹ at a frequency of 15 THz while the minimum doping value approaches zero from Fig. 4C to Fig. 4E (Fig. 4B).

Our calculations show that as the metasurfaces tend to the singular case, the transmissivity peaks are crushed closer together, finally merging into a continuum. Every point in the transmission spectrum defines a hidden wave vector, k_h, through the dispersion relationship ω(k_h). In this way, a discrete spectrum with absorption peaks at the Wood’s anomaly positions is broadened into a continuum. One layer of a singular graphene metasurface can reject almost half of the terahertz radiation incident on it over a broadband width.

Some qualifications should be made. We have assumed a local form for ε, ignoring that at very short distances this assumption breaks down (12–14), which will remove the possibility of a perfect singularity as postulated here. Systems also have resistive loss as we have discussed, and to some extent, the two effects compensate for one another. Nonlocality by removing the perfect singularity will tend to produce a discrete spectrum; however, loss will smear a discrete but still dense spectrum into a continuum, leaving the broadband absorption intact. This is shown in Fig. 4E, where the green line corresponds to a lower mobility of 5 × 10³ cm² V⁻¹ s⁻¹.

The model system described here is a realization of the exotic phenomenon of a compacted dimension. The hidden dimension has a dramatic effect on electromagnetic properties of the system: Transmission through a single sheet of graphene structured according to our prescription shows strong broadband absorption of terahertz radiation, as opposed to the isolated absorption peaks of a conventional grating.

REFERENCES AND NOTES

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Compacted dimensions and singular plasmonic surfaces
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How to hide a dimension from view
Compacted dimensions are essential ingredients of advanced string theories; the extra dimensions used to describe the world are hidden from view, or compacted. Pendry et al. used concepts of transformation optics, in which metamaterials are designed to produce specific optical properties, to show theoretically that compacted dimensions could be possible in plasmonic structures. For example, simulations with a metallic grating and periodically doped graphene revealed that two-dimensional structures exhibit optical properties of a bulk material. To all intents and purposes, it looks as if one of the dimensions is compacted. The results could have practical application in designing broadband optical absorbers.

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